

# The Energy Criterion and Dynamical Symmetry Breaking in a $SU(3)_L \otimes U(1)_X$ Extension of the Standard Model

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## Abstract

The coupling constants  $g_L$  and  $g_X$  of some versions of the  $SU(3)_L \otimes U(1)_X$  extension of the standard model are related through to relationship  $g_X^2/g_L^2 = \sin^2 \theta_w / (1 - 4 \sin^2 \theta_w)$ . This fact suggest that the  $SU(3)_L \otimes U(1)_X$  gauge symmetry in this class of models can be broken dynamically to the standard model at TeV scale without requiring the introduction of fundamental scalars. This possibility was investigated by Das and Jain who considered only the first version of this class of models. In this brief report we discuss an energy criterion to verify the most probable version of the  $SU(3)_L \otimes U(1)_X$  model that is realized in nature.

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The Standard Model of elementary particles is in excellent agreement with the experimental data and has explained many features of particle physics along the years. Despite the success there are some points in the model as, for instance, the flavor problem or the enormous range of masses between the lightest and heaviest fermions and other peculiarities the could be better explained with the introduction of new fields and symetries. One of the possibilities in this direction is to assume an extension of the Standard Model based on  $G_{3n1} \equiv SU(3)_C \otimes SU(n)_L \otimes U(1)_X$  [1, 2, 3], where  $n = 3, 4$ . This class of the models predicts interesting new physics at TeV scale[4] and address some fundamental questions that does not can be explained in the framework of the Standard Model. As a brief example we can mention the flavor problem[5] and the question of electric charge quantization[6].

One interesting feature of some versions of these models[1, 2] is the following relationship among the coupling constants  $g_L$  and  $g_X$  associated to the gauge group  $SU(3)_L \otimes U(1)_X$

$$\frac{\alpha_X}{\alpha_L} = \frac{\sin^2(\theta_w)}{1 - 4 \sin^2(\theta_w)} \quad (1)$$

where  $\alpha_i = g_i^2/4\pi$ , with  $i = X, L$  and  $\sin(\theta_w)$  is the electroweak mixing angle. Then, in a high energy scale, when  $\sin^2(\theta_w)(\mu) \approx 1/4$ , the coupling constant  $g_X^2$  becomes very strong. The energy scale where the theory becomes non-perturbative may be estimated as being of order few TeVs, and this fact suggest that the gauge symmetry  $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$  of this class of models maybe be broken to  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  without requiring the introduction of fundamental scalars.

In Ref[7] the authors investigated this possibility, however, in that work only the first version of these models was considered [1]. As we emphasize in the text, there are other versions of 3-3-1 models that have the relationship show in Eq.(1) among the coupling constants  $\alpha_X$  and  $\alpha_L$ [2]. In this work we discuss an energy criterion[8] to select the most probable version of the 3-3-1 model. We show that just only version[2] of these models lead to a deeper minimum of the effective potential. Therefore, only in this version the coupling constant  $\alpha_X$  would be strong enough, and energetically preferred, in order to promote the dynamical symmetry breaking.

We will begin writing the Schwinger-Dyson equation for quarks considering only the  $U(1)_X$  interaction once this is the dominant contribution

$$S^{-1}(p) = \not{p} - i \int \frac{d^4 q}{(2\pi)^4} \Gamma_\mu(p, q) S(q) \Gamma_\nu D_{M_{Z'}}^{\mu\nu}(p - q) \quad (2)$$

where we assumed the rainbow approximation for the vertex  $\Gamma_{\mu,\nu}$ , with  $\Gamma_{\mu,\nu} = g_V \gamma_{\mu,\nu} + g_A \gamma_{\mu,\nu} \gamma_5$ ,  $g_V = g_X^2 (X_L + X_R)/2$  and  $g_A = g_X^2 (X_R - X_L)/2$ .  $X_L$  and  $X_R$  are respectively the  $U(1)_X$  charges attributed to the chiral components of the exotic quarks  $J_{1L}$  and  $J_{1R}$ .

With the purpose of simplifying the calculations is convenient choose the Landau gauge. In this case the  $Z'$  propagator can be written in the following form

$$iD_{M_{Z'}}^{\mu\nu}(p-q) = -i \frac{[g_{\mu\nu} - (p-q)_\mu(p-q)_\nu/(p-q)^2]}{(p-q)^2 - M_{Z'}^2}.$$

Writing the quark propagator as  $iS^{-1}(p) = i(\not{p} - \Sigma(p^2))$ , and considering the equation above, we can write

$$\Sigma(p^2) = -ia \int d^4q \frac{\Sigma(q^2)}{[q^2 - \Sigma^2(q^2)]} \frac{1}{[(p-q)^2 - M_{Z'}^2]} \quad (3)$$

where  $a = \frac{3g_X^2 X_L X_R}{(2\pi)^4}$ .

The dynamical mass generated for the  $Z'$  boson can be estimated as  $M_{Z'} \sim \mu_X$ , where  $\mu_X \sim O(TeV)$  is the energy scale where the  $U(1)_X$  interaction becomes sufficiently strong to break dynamically the quiral and gauge symmetries. The Eq.(3) is one nonlinear integral equation and can be reduced to a nonlinear differential equation in momentum space which can be solved only numerically. However, we can use the following linearized version of this last one

$$\frac{d}{dp^2} \left[ (p^2 + \mu_X^2)^2 \frac{d\Sigma(p^2)}{dp^2} \right] = -a \frac{p^2 \Sigma(p^2)}{(p^2 + \mu_X^2)} \quad (4)$$

as a good approximation, where we substituted in the denominator  $\Sigma^2(p^2)$  by  $\mu_X^2$ . Once only the exotic quarks  $J$  will acquire mass at this scale the dynamical mass generated for such particles will be of the same order that the mass generated for the new gauge bosons ( $V^\pm, Z'$ ), justifying our approximation. The most general solution for this equation can be written as

$$\Sigma(p^2) = \frac{f(p^2)}{2n} [C_n J_n[f(p^2)] \Gamma(n) - C_m J_m[f(p^2)] \Gamma(m)] \quad (5)$$

where  $J_{n,m}[z]$  are Bessel functions,  $\Gamma(n, m)$  is the Gamma function and  $C_{n,m}$  are constants of integration. For convenience, we defined

$$f(p^2) = \left( \frac{4a\mu_X^2}{p^2 + \mu_X^2} \right)^{\frac{1}{2}}$$

with  $n = -m \equiv \sqrt{1 - 4a}$ . Eq.(5) has two asymptotic solutions

$$\Sigma(p^2)_1 \sim \frac{\mu_X^3}{p^2} \left( \frac{p^2}{\mu_X^2} \right)^a \quad (6)$$

$$\Sigma(p^2)_2 \sim \mu_X \left( \frac{p^2}{\mu_X^2} \right)^{-a}. \quad (7)$$

which are named in the literature respectively as Regular and Irregular solutions[9, 10]. Considering the running of the  $U(1)_X$  coupling constant

$$\alpha_X(p^2) = \frac{\alpha_X(\mu^2)}{1 + \alpha_X(\mu^2)b_X \ln(\mu^2/p^2)} \quad (8)$$

where  $b_X \equiv \Sigma X^2/6\pi$ , the asymptotic solutions of the Eq.(6) and (7) can be written as

$$\Sigma(p^2)_1 \sim \frac{\mu_X^3}{p^2} \left( \frac{\alpha_X(p^2)}{\alpha_X(\mu_X^2)} \right)^c \quad (9)$$

$$\Sigma(p^2)_2 \sim \mu_X \left( \frac{\alpha_X(p^2)}{\alpha_X(\mu_X^2)} \right)^{-c}. \quad (10)$$

where  $c = \frac{9X_L X_R}{2\Sigma X^2}$  and in the expressions above  $\Sigma X^2$  is the sum of the square of the  $U(1)_X$  charges of the models to be considered in this work. There is an restriction about the Irregular solution, Eq.(10). For this solution it is necessary that  $c > 1/2$  [10]. If we consider the formal equivalence between the solution of the Schwinger-Dyson equation with the Bethe-Salpeter one for pseudo-scalar bound states, the above restriction indicates the condition for wave function normalization of the Goldstone bosons. In all the models that we will be considered in this work we have  $c < 1/2$ , and for this reason we will just consider the fermionic self-energy Eq.(9). In the paragraph below we will compute the vacuum energy for this fermionic self-energy making use of the effective potential for composite operators.

The effective potential for composite operators is given by the following expression[11]

$$V(S, D) = -i \int \frac{d^4 p}{(2\pi)^4} Tr(\ln S_0^{-1} S - S_0^{-1} S + 1) + V_2(S, D), \quad (11)$$

where in this expression  $S$  and  $D$  are the complete propagators of fermions and gauge bosons and  $S_0$ ,  $D_0$ , are the corresponding bare propagators. The function  $V_2(S, D)$  is given by the two-particle irreducible vacuum diagram depicted in the figure 1.

The expression for  $V_2(S, D)$  can be represented analytically in the Hartree-Fock approximation by the following equation

$$iV_2(S, D) = -\frac{1}{2} Tr(\Gamma S \Gamma S D) \quad (12)$$

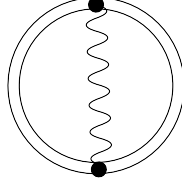


FIG. 1: Diagramm for two loops ( $V_2$ ) contribution to the effective potential

where for simplicity in this equation we have not written the gauge and Lorentz indices, as well as the momentum integrals and we are representing the fermion proper vertex by  $\Gamma$ .

We want to determine numerically the vacuum expectation value for the fermionic self-energy given by the Eq.(9), for the models described in the Ref.[1](A,B) and Ref.[2](C). However, it is better to compute the vacuum energy density, which is given by the effective potential calculated at minimum subtracted by its perturbative part which does not contribute to dynamical mass generation[11, 12]

$$\langle \Omega \rangle = V_{min}(S, D) - V_{min}(S_p, D_p), \quad (13)$$

where we indicate in the above expression the perturbative counterpart of  $S$  and  $D$  respectively by  $S_p$ ,  $D_p$ .  $V_{min}(S, D)$  is obtained substituting the SDE, Eq.(2), in the Eq.(11) and we can verify that in the chiral limit  $S_p = S_0$ . The complete fermion propagator  $S$  is related to the free propagator by the equation  $S^{-1} = S_0^{-1} - \Sigma$ , with  $S_0 = i/\not{p}$  and we chose to work in the Landau gauge. After a Euclidean rotation, we find that  $\Omega_{min} \equiv \langle \Omega \rangle$  is equal to[12]

$$\Omega_{min} = -2 \int \frac{d^4 p}{(2\pi)^4} V(p^2, \Sigma) \quad (14)$$

where we have defined the function  $V(p^2, \Sigma)$  as

$$V(p^2, \Sigma) = \left[ \ln\left(\frac{p^2 + \Sigma^2}{p^2}\right) - \frac{\Sigma^2}{p^2 + \Sigma^2} \right].$$

We can still expand  $\Omega_{min}$  in powers of  $\Sigma^2/p^2$ , so that

$$\Omega_{min} \approx - \int \frac{d^4 p}{(2\pi)^4} \frac{\Sigma^4}{p^4}. \quad (15)$$

To obtain an analytical formula for the vacuum energy density we will consider the substitution  $x \rightarrow \frac{p^2}{\mu_x^2}$  in the Eqs.(9) and (15), and we will assume the following Mellin transform[13]

$$[1 + \kappa \ln x]^{-\epsilon} = \frac{1}{\Gamma(\epsilon)} \int_0^\infty d\sigma e^{-\sigma} (x)^{-\sigma\kappa} \sigma^{\epsilon-1} \quad (16)$$

that will simplify considerably the calculation. In this Mellin transform we identified  $\kappa = -\alpha_X b_X$  and  $\epsilon = 4c$ . Then, after we substitute Eq.(9) in to Eq.(15), and perform the integration we obtain

$$\Omega_{min} \approx -\frac{\mu_X^4 \zeta}{32\pi^2} \left[ 1 + \frac{1}{8\pi^2 \zeta} \frac{\Sigma X^2}{X_L X_R} + O\left(\frac{1}{\zeta(\Sigma X^2)^2}\right) \dots \right]. \quad (17)$$

Where  $\zeta \equiv \left(1 + \frac{3}{4\pi}\right)$ , and to obtain this last equation we made use of the scaling law  $\frac{g_X^2 X_L X_R}{4\pi} \approx 1$  [14].

In table I we present the weak hypercharge content attributed to each model and the respective value obtained for the minimum of the potential. As it is possible to verify the deepest minimum of energy happens only for the model C because at scale  $\mu_X$  this model take to  $U(1)_X$  coupling much more stronger and close to the critical value  $\alpha_c$  necessary to promote the dynamical symmetry breaking.

As the authors of Ref.[7] argued, the gauge symmetry breaking of  $SU(3)_L \otimes U(1)_X$  in 3-1 models can be implemented dynamically because the scale of a few TeVs,  $\mu_X$ , the  $U(1)_X$  coupling constant becomes strong as we approach the peak existent in Eq.(1). The exotic quarks  $J$  introduced in these models will form a condensate  $\langle \bar{J}J \rangle$  breaking  $SU(3)_L \otimes U(1)_X$  to  $SU(2)_L \otimes U(1)_X$  at this scale.

The electroweak symmetry could be broken dynamically by a top condensate[15]. In this case, as Das and Jain argued, it would be necessary to introduce new exotic quarks  $\chi_L$  and  $\chi_R$  in the model in order to maintain the top quark mass around  $170 GeV$ , which is an interesting possibility that we intend to explore in the future.

In this work we show that just one version[2] of this class of models lead to a deeper minimum of the effective potential, establishing a criterion for the choice of the most probable version of the  $SU(3)_L \otimes U(1)_X$  model that is realized in nature.

After the integration of equation (??), we obtain the value for the vacuum energy density(minimum of energy),  $\Omega_{min}$ , for models of the type  $A$  to  $C$ , as shown in table I. At the scale  $\mu_X$ , the coupling constant  $\alpha_X$  of the  $U(1)_X$  group becomes strong enough to promote the dynamical symmetry breaking of the model. However, this happens only for the version C, which is the one that corresponds to the deepest state of energy.

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$32\pi^2\Omega_{min}/\mu_X^4$	Model	$U(1)_X$ Charges
-1.535	A,B	leptons:
		$X_{l_{aL}} = 0$
		quarks:
		$X_{Q_{1L}} = 2/3$ , $X_{u_{1R}} = 2/3$
		$X_{d_{1R}} = -1/3$ , $X_{J_{1R}} = 5/3$
-1.605	C	$X_{Q_{iL}} = -1/3$ , $X_{u_{iR}} = 2/3$
		$X_{d_{iR}} = -1/3$ , $X_{J_{iR}} = -4/3$
		leptons:
		$X_{l_{aL}} = 0$ , $X_{l_{aR}} = -1$
		$X_{E_{aR}} = 1$
-1.605	C	quarks:
		$X_{Q_{1L}} = 2/3$ , $X_{u_{1R}} = 2/3$
		$X_{d_{1R}} = -1/3$ , $X_{J_{1R}} = 5/3$
		$X_{Q_{iL}} = -1/3$ , $X_{u_{iR}} = 2/3$
		$X_{d_{iR}} = -1/3$ , $X_{J_{iR}} = -4/3$

TABLE I: In the above table  $i = 2, 3$  labels the second and third quark families and  $a = 1..3$ .  $X_{l_{aL}}$ ,  $X_{Q_{1L}}$  and  $X_{Q_{iL}}$  represent, respectively, the hipercharges attributed to  $\mathbf{3}$  and  $\mathbf{3}^*$  of the leptons( $l$ ) e quarks( $Q$ ). We also show the hipercharge content attributed to the models A,B and C. The fermionic content associated to the model B is the same as in model A, however, it is the third quark generation that will transform as  $\mathbf{3}^*$ , the first and second generation will transform as  $\mathbf{3}$ .

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